1 COMPLEX NUMBERS

1. INTRODUCTION TO IOTA AND COMPLEX NUMBERS

	♦	INTEGRAL POWER OF IOTA, INTRODUCTION TO COMPLEX NUMBERS
	♦	REAL AND IMAGINARY PARTS OF COMPLEX NUMBERS

SYNOPSIS-1

"Complex number is the combination of real and imaginary"

BASIC CONCEPTS OF COMPLEX NUMBER:

1) Definition: A number of the form x + iy, where x, y $\in$ R and i = $\sqrt{-1}$ is called a complex number and 'i' is called iota.

A complex number is usually denoted by z and the set of complex number is denoted by C.

i.e., C = {x + iy : x$\in$ R, y $\in$ R, i = $\sqrt{-1}$ }

For example, 5 + 3i, -1 + I, 0 + 4i, 4 + 0i etc. are complex numbers.

i) Euler was the first mathematician to introduce the symbol i (iota) for the square root of -1 with property $i^2 = -1$. He also called this symbol as the imaginary unit.

ii) For any positive real number 'a', we have $\sqrt{-a} = \sqrt{-1 \times a} = \sqrt{-1}\sqrt{a} = i\sqrt{a}$

iii) The property $\sqrt{a}\sqrt{b} = \sqrt{ab}$ is valid only if at least one of a and b is non-negative. If a and b are both negative, then $\sqrt{a}\sqrt{b} = -\sqrt{|a|.|b|}$.

2) Integral powers of iota (i): Since i = $\sqrt{-1}$ hence we have $i^2 = -1$, $i^3 = -i$ and $i^4 = 1$. To find the value of i^n (n>4), first divide n by 4. Let q be the quotient and r be the remainder.

i.e., n = 4q+r where $0 \leq r \leq 3$

$i^n = i^{4q+r} = (i^4)^q = (i^4)^q.(i)^r = i^r$

In general, we have the following results $i^{4n} = 1$, $i^{4n+1} = i$, $i^{4n+2} = -1$, $i^{4n+3} = -i$, where n is any integer.

Illustration -1 $\sqrt{-2}\sqrt{-3} =$

Solution (1) $\sqrt{-2}\sqrt{-3} = i\sqrt{2}\,i\sqrt{3} = i^2\sqrt{6} = -\sqrt{6}$

Illustration -2 If n is a positive integer, then which of the following relations is false

Solution (2) We know that $i^2 = -1$

$\Rightarrow \left(i^2\right)^2 = \left(-1\right)^2 = 1 \Rightarrow i^{4n} = 1^n$ and therefore $i^{4n-1} = -i$

Illustration -3 If $i = \sqrt{-1}$, then $1 + i^2 + i^3 - i^6 + i^8$ is equal to

Solution (3) $1 + i^2 + i^3 - i^6 + i^8 = 1 - 1 - i + 1 + 1 = 2 - i$

REAL AND IMAGINARY PARTS OF A COMPLEX NUMBER

If x and y are two real numbers, then a number of the form $z = x + iy$ is called a complex number. Here 'x' is known as the real part of z and 'y' is known as the imaginary part of z. The real part of z is denoted by Re(z) and the imaginary part by Im(z).

If $z = 3 - 4i$, then Re (z) = 3 and Im(z) = -4

A complex number z is purely real if its imaginary part is zero i.e., Im(z) = 0 and purely imaginary if its real part is zero i.e., Re(z) = 0.

Illustration -1 The imaginary part of i^i is

Solution (1) $A = i^i$ $\log A = \log i^i \Rightarrow \log A = i \log i$ $\log A = i \log (0 + i)$

$\Rightarrow \log_e A = i\log(\cos \pi/2 + i\sin \pi/2) = i\log_e e^{i\pi/2} = i^2 \pi/2$

$\Rightarrow \log_e A = -\pi/2$

$\Rightarrow A = e^{-\pi/2}$

Therefore, imaginary part is 0.

Illustration - 2 If z_1 and z_2 be two complex number, then Re $(z_1 z_2)$

Solution (4) If z_1 and z_2 be two complex number then Re $(z_1 z_2)$ = Re (z_1) Re (z_2) – Im (z_1) Imz(z_2)

1. INTRODUCTION TO IOTA AND COMPLEX NUMBERS

WORK SHEET LEVEL–I MAINS CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

INTEGRAL POWER IOTA, INTRODUCTION TO COMPLEX NUMBERS

1. A number of the form x +iy, where $x, y \in R$ and $i = \sqrt{-1}$ is called a complex number
   1) True 2) False 3) Doubtful 4) can't say
2. In complex number x+iy, i is called
   1) Input 2) Inner space 3) Iota 4) Inlet
3. The mathematician who introduced the symbol 'i' is
   1) Newton 2) Euler 3) Fleming 4) Ramanujan
4. If n is any integer, then which of the following is true
   1) $i^{4n} = 1, i^{4n+1} = i$ 2) $i^{4n+2} = -1$ 3) $i^{4n+3} = -i$ 4) All the above

REAL PART AND IMAGINARY PART OF COMPLEX NUMBERS

5. In the complex number z = x + iy,
   1) x represents real part of z and is denoted by Re (z)
   2) y represents imaginary of z and is denoted by Im(z)
   3) Both (1) and (2) are true
   4) None
6. A complex number is purely real if its
   1) Imaginary part is zero
   2) Real part is zero
   3) Both real and imaginary parts are zero
   4) Only real part is zero

LEVEL–II

INTEGRAL POWER IOTA, INTRODUCTION TO COMPLEX NUMBERS

7. If $i = \sqrt{-1}$, then $1 + i^2 + i^3 + i^3 - i^6 + i^8$ is equal to
   1) 2 – i 2) 1 3) 3 4) –1
8. The value of $(1+i)^5 \times (1-i)^5$ is
   1) –8 2) 8i 3) 8 4) 32
9. The value of $\frac{i^{592}+i^{590}+i^{588}+i^{586}+i^{584}}{i^{582}+i^{580}+i^{578}+i^{576}+i^{574}} - 1 =$
   1) –1 2) –2 3) –3 4) –4
10. $1 + i^2 + i^4 + i^6 + \ldots + i^{2n}$ is
    1) Positive 2) Negative
    3) Zero 4) Cannot be determined

11. $i^2 + i^4 + i^6 + \ldots\ldots$.up (2n+1) terms =

1) i 2) –i 3) 1 4) –1

REAL PART AND IMAGINARY PART OF COMPLEX NUMBERS

12. If x, $y \in R$, then x+iy is a non-real complex number, if

1) x = 0 2) y = 0 3) $x \neq 0$ 4) $y \neq 0$

13. $\text{Re}\left[\frac{(1+i)^2}{3-i}\right] =$

1) –1/5 2) 1/5 3) 1/10 4) –1/10

LEVEL-III ADVANCED CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

14. The complex number $\frac{1+2i}{1-i}$ lies in which quadrant of the complex plane

1) First 2) Second 3) Third 4) Fourth

15. Let $i^2 = -1$. Then $\left(i^{10} - \frac{1}{i^{11}}\right) + \left(i^{11} - \frac{1}{i^{12}}\right) + \left(i^{12} - \frac{1}{i^{13}}\right) + \left(i^{13} - \frac{1}{i^{14}}\right) + \left(i^{14} - \frac{1}{i^{15}}\right) =$

1) –1+i 2) –1– i 3) 1+ i 4) –i

16. The value of the sum $\sum_{n=1}^{13}\left(i^n + i^{n+1}\right)$, where $i = \sqrt{-1}$, equals

1) I 2) i – 1 3) –i 4) 0

LEVEL-IV

STATEMENT TYPE QUESTIONS

17. Statement I: If 3 + 4i is complex number 3 is real part and 4 is imaginary part

Statement II: x + iy, complex number, x is real part, y is imaginary

1) Both the statements are true

2) Both the statements are false

3) Statement I is true, statement II is false

4) Statement I is false, statement II is true.

MULTI CORRECT ANSWER TYPE QUESTIONS

18. Which of the following is/are true

1) The set of complex number is indented by z

2) $i^2 = -1$

3) $i^3 = -i$

4) $i^4 = 1$

19. Which of the following is /are false

1) Every real number is complex number

2) Every complex number is real number

3) $i^2 = 1$

4) A, B, C are true

INTEGER TYPE QUESTIONS

20. If the value of $i^{242} = -k$ then k = ________.

21. If the value of $i^{469} = mi$, then m = ______.

22. If the simplified value of $i^{57} + \frac{1}{i^{125}}$ is = p-1 the p =

LEVEL-V

COMPREHENSION TYPE QUESTIONS

PASSAGE

$i^2 = -1$ then

23. $\left(\frac{1+i}{1-i}\right)^4 + \left(\frac{1-i}{1+i}\right)^4 =$

1) 0 2) 1 3) 2 4) 4

24. $\frac{(1+i)^{2011}}{(1-i)^{2009}} =$

1) –1 2) 1 3) 2 4) –2

25. $\frac{(1+i)^{2016}}{(1-i)^{2014}} =$

1) –2i 2) 2i 3) 2 4) –2

MATRIX MATCH TYPE QUESTIONS

26.

COLUMN-I	COLUMN-II
a) $i^7 =$	p) 1
b) $i^8 =$	q) –1
c) i^9	r) –i
d) i^{10}	s) i

2. ALGEBRAIC OPERATIONS WITH COMPLEX NUMBERS

	♦	ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION OF COMPLEX NUMBERS
	♦	EQUALITY OF TWO COMPLEX NUMBERS

ALGEBRAIC OPERATIONS WITH COMPLEX NUMBERS

Let two complex numbers $z_1 = a + ib$ and $z_2 = c + id$

Addition $(z_1 + z_2)$: $(a+ib) + (c+id) = (a+c) + i(b+d)$

Subtraction $(z_1 - z_2)$: $(a+ib) - (c+id) = (a-c) + i(b-d)$

Multiplication $(z_1.z_2)$: $(a+ib) \times (c+id) = (ac - bd) + i(ad + bc)$

Division (z_1 / z_2) : $\frac{a+ib}{c+id}$

(where at least one of c and d is non-zero)

$$\frac{a+ib}{c+id} = \frac{(a+ib)}{(c+id)} \cdot \frac{(c-id)}{(c-id)} \text{ (Rationalization)}$$

$$\frac{a+id}{c+id} = \frac{(ac+bd)}{c^2+d^2} + \frac{i(bc-ad)}{c^2+d^2}$$

Properties of algebraic operations on complex number: Let z_1, z_2 and z_3 are any three complex numbers then

i) Addition of complex numbers satisfies the commutative and associative properties
i.e., $z_1 + z_2 = z_2 + z_1$ and $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$.

ii) Multiplication of complex numbers satisfies the commutative and associative properties.
i.e., $z_1z_2 = z_2z_1$ and $(z_1z_2)z_3 = z_1(z_2z_3)$.

iii) Multiplication of complex number is distributive over addition
i.e., $z_1(z_2+z_3) = z_1z_2 + z_1z_3$ and $(z_2+z_3)\, z_1 = z_2z_1 + z_3z_1$.

Illustration-1 $\left(\frac{1}{1-2i}+\frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)=$

Solution (4) $\left(\frac{1}{1-2i}+\frac{3}{1+i}\right)\left(\frac{3+4i}{2-4i}\right)$

$$=\left[\frac{1+2i}{1^2+2^2}+\frac{3-3i}{1^2+1^2}\right]\left[\frac{6-16+12i+8i}{2^2+4^2}\right]$$

$$=\left(\frac{2+4i+15-15i}{10}\right)\left(\frac{-1+2i}{2}\right)$$

$$=\frac{(17-11i)(-1+2)}{20}=\frac{5+45i}{20}=\frac{1}{4}+\frac{9}{4}$$

EQUALITY OF TWO COMPLEX NUMBERS

Two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are said to be equal if and only if their real and imaginary parts are separately equal.

i.e., $z_1 = z_2 \Leftrightarrow x_1 + iy_1 = x_2 + iy_2 \Leftrightarrow x_1 = x_2$ and $y_1 = y_2$.

Complex number do not possess the property of order i.e., (a + ib) < (or) > (c+id) is not defined. For example, the statement (9 + 6i) > (3 + 2i) makes no sense.

Illustration-1 The statement (a+ib) < (c + id) is true for

Solution (4) a + ib< c + id, defined if and only if its imaginary parts must be equal to zero, i.e. b = d = 0. So, $b^2 + d^2 = 0$

2. ALGEBRAIC OPERATIONS WITH COMPLEX NUMBERS

WORK SHEET LEVEL–I MAINS CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

ADDITION, SUBSTATION, MULTIPLICATION AND DIVISION OF COMPLEX NUMBERS

1. If $z_1 = a + ib$, $z_2 = c + id$, the which of the following is correct
   1) $z_1 + z_2 = (a+c) + i(b+d)$
   2) $z_1 - z_2 = (a-c + i(b-d)$
   3) $z_1 - z_2 = (a-c) + i(b-d)$
   4) Both (1) and (2) are true

2. If $z_1 = a + ib$, $z_2 = c + id$ then
   1) $z_1 \times z_2 = (ac - bd) + i(ad + bc)$
   2) $z_1/z_2 = \left(\frac{ac+bd}{c^2+d^2}\right) + \frac{i(bc-ad)}{c^2+d^2}$
   3) Both (1) and (2) are false
   4) Both (1) and (2) are true

3. Which of the following is true
   1) Addition of complex numbers satisfies the commutative and associative
   2) Multiplication of complex numbers satisfies the commutative and associative properties
   3) Multiplication of complex numbers is distribution
   4) All the above

EQUALITY OF TWO COMPLEX NUMBERS

4. If $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$ and if $z_1 = z_2$ then
   1) $x_1 = x_2$
   2) $y_1 = y_2$
   3) $\begin{matrix} x_1 \neq x_2 \\ y_1 \neq y_2 \end{matrix}$
   4) Both (1) and (2) are true

LEVEL–II

ADDITION, SUBSTATION, MULTIPLICATION AND DIVISION OF COMPLEX NUMBERS

5. The sum of $3 + 4i$ and $2 - 3i$ is
   1) $1 + 7i$ 2) $5 + i$ 3) $3 - 2i$ 4) $2 + 4i$

6. The additive inverse of $(-3 + 4i)$ is
   1) $3 + 4i$ 2) $3 - 4i$ 3) $8 + 4i$ 4) $-3 + 4i$

7. If $z_1 = -2 + i$, $z_2 = -2 - i$ then $z_1 - z_2$ is ______.
   1) $2i$ 2) 0 3) 4 4) -4

8. If $z_1 = 1 + i$, $z_2 = 1 - i$, then $z_1 z_2$ is _____
   1) 0 2) 2 3) -2 4) 1

9. If $z_1 = 1 + i$, $z_2 = 1 - i$, then $\frac{Z_1}{Z_2}$ is _______.
   1) $-i$ 2) 1 3) 0 4) i

EQUALITY OF TWO COMPLEX NUMBERS

10. If $A = a + ib$ and $B = 2 + 3i$ if $A = B$ then the value of 'b' is
    1) 3 2) 2 3) 5 4) 1

11. If $\frac{5(-8)+6i}{(1+i)^2}$ = a + ib, then (a, b) equals

1) (15, 20) 2) (20, 15) 3) –15, 20) 4 None of these

LEVEL-III ADVANCED CORNER

SINGLE CORRECT ANSWER TYPE QUESTIONS

12. Real part of $\frac{a+ib}{a-ib}$ is

1) $\frac{2ab}{a^2+b^2}$ 2) $\frac{a^2-b^2}{a^2+b^2}$ 3) $\frac{a^2+b^2}{a^2-b^2}$ 4) $\frac{2ab}{a^2-b^2}$

13. If (1 – i) (1 + 2i) (1 – 3i) = x + iy then (x, y) =

1) (1, –6) 2) (6, 8) 3) (6, –8) 4) (8, –6)

14. If $\left(\frac{1+i}{1-i}\right)^5 - \left(\frac{1-i}{1+i}\right)^5$ = A + iB then A, B are

1) 0, 2 2) 0, –2 3) 2, 0 4) –2, 0

LEVEL-IV

STATEMENT TYPE QUESTIONS

15. Statement I: The value of i^{460} is – 1
Statement II: The value of $(1 + i + i^2 + i^3 + i^4 + i^5)(1 + i) = 2i$

1) Only I is true 2) Only II is true
3) Only I is true 4) Neither I nor II are true

16. Statement I: $\left(\frac{1+i}{\sqrt{2}}\right)^8 + \left(\frac{1-i}{\sqrt{2}}\right)^8$ is 2

Statement II: The value of $\left(\frac{\sqrt{3}+i}{2}\right)^6 + \left(\frac{i-\sqrt{3}}{2}\right)^6$ is – 2

1) Only I is true 2) Only II is true
3) Both I and II are true 4) Neither I nor II are true

MULTI CORRECT ANSWER TYPE QUESTIONS

17. If $(1 - i)^3 (1 + i)$ = a + ib then a, b are

1) 0 2) –2 3) 4 4) –4

18. If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ = x + iy then

1) x = 0 2) y = –2 3) x = 2 4) y = 0

INTEGER TYPE QUESTIONS

19. If $\left(\frac{1+i}{1-i}\right)^x$ =1, then x = _______ times of n, n is natural number.

20. If $\sum_{k=0}^{100} i^k$ = x + iy, then the values of x + y = ________

LEVEL-V

COMPREHENSION TYPE QUESTIONS

PASSAGE

Using the concept of imaginary numbers i.e. $i=\sqrt{-1}$ or i^2 = 1, we can simplify the values of some expressions

For example, if x = 2 – 3i then we can find the $x^2 - 4x + 13$ as:

X = 2 – 3i

$\Rightarrow x-2=-3i$

$\Rightarrow (x-2)^2=(-3i)^2$

$\Rightarrow x^2-4x+4=9i^2$

$\Rightarrow x^2-4x+4=-9$

$\Rightarrow x^2-4x+13=0$

Use the above information to solve the questions given below

21. If $x=\frac{-5+i\sqrt{3}}{2}$ then the value of $(x^2 + 5x)^2 + x(x + 5)$ is ________.

1) 40 2) 42 3) 44 4) 38

22. If x = 3 – 5i then $x^3 - 10x^2 + 58x - 136$ =

1) 0 2) I 3) –I 4) 1

23. If x = – 5 + 4i then $x^4 + 9x^3 + 35x^2 - x + 4$ =

1) –160 2) 160 3) –170 4) 170

MATRIX MATCH TYPE QUESTIONS

24.

COLUMN-I	COLUMN-II
a) $(1 + i)^3 (1 - i)^3$	p) 0
b) $\left(\frac{1+i}{1-i}\right)^2-\left(\frac{1-i}{1+i}\right)^2=$	q) 8
c) $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3=$	r) 2i
d) $\left(\frac{1+i}{1-i}\right)^4-\left(\frac{1-i}{1+i}\right)^4=$	s) – 2i